Projectile Motion

- A projectile is an object that is in free fall and subject only to gravity and air resistance
- Often, air resistance is small and we ignore it (e.g., long jump, high jump, or free throw)
- However, sometimes air resistance should not be ignored (e.g., golf or tennis)
- If air resistance is ignored, projectiles are an example of constant acceleration

Galileo's equations of constant acceleration:

$$v_2 = v_1 + at$$
$$d = v_1 t + ½at^2$$
$$v_2^2 = v_1^2 + 2ad$$

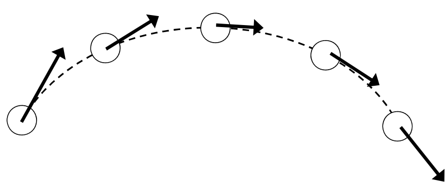
d = displacement
v = velocity
a = acceleration
t = time
subscript 1 = initial time
subscript 2 = final time

You may also use the equations below. They are derivations of the equations above and they may save you time, but they are not absolutely necessary

$$\text{Maximal Height} = \frac{(V_1)^2}{-2g} \qquad \text{Time of Flight} = \frac{(2V_1)}{-g} \qquad \text{Time of Flight} = \frac{-V_1 - \sqrt{V_1^2 - 2gh}}{g}$$

Projectile Motion

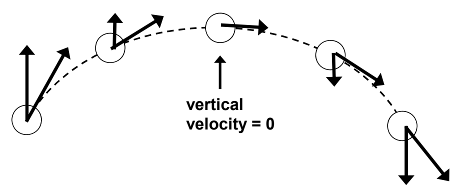
- If air resistance is negligible, projectile motion is simply a special case of uniform acceleration
- With no air resistance, the path followed by a projectile will be a parabola



Vertical Velocity

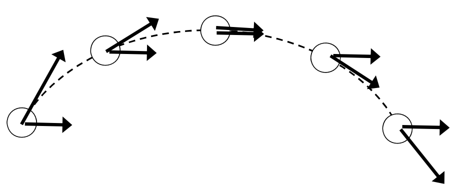
Horizontal and vertical components of projectile motion are independent and are treated separately

Gravity will cause the vertical component of velocity to change during the flight



Horizontal Velocity

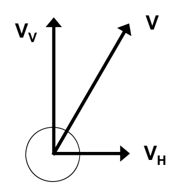
The horizontal component of velocity will be constant for the whole flight. But, why?



Projectile Motion

In review, if air resistance is negligible:

- Horizontal velocity does not change while the object is in the air
- Vertical velocity changes by -9.81 m/s, each second the object is in the air

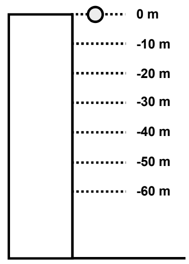


Vertical Motion of a Projectile

Drop a water balloon from the edge of a building, and it's vertical motion will be very predictable

Use: $d = v_1t + ½at^2$
$v_2 = v_1 + at$

time	position	velocity	acceleration
0 s			
1 s			
2 s			
3 s			

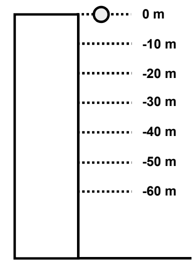


Vertical Motion of a Projectile

Drop a water balloon from the top of a building, and the ballon's vertical motion will be very predictable

Use: $d = v_1t + ½at^2$
$v_2 = v_1 + at$

time	position	velocity	acceleration
0 s	0 m	0 m/s	−9.8 m/s²
1 s			
2 s			
3 s			



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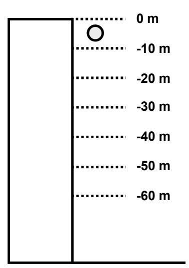
2 s			
3 s			



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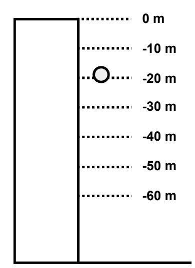
1 s	−4.9 m	−9.8 m/s	−9.8 m/s²
2 s	−19.6 m	−19.6 m/s	−9.8 m/s²
3 s			



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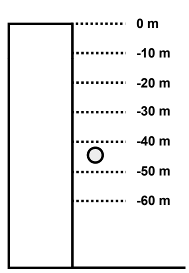
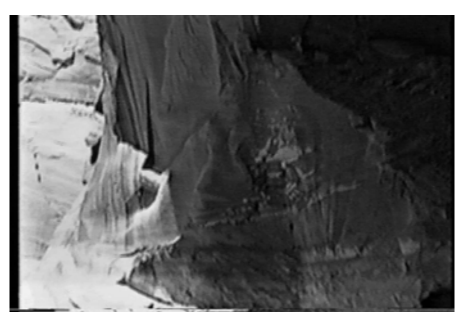
1 s	−4.9 m	−9.8 m/s	−9.8 m/s²
2 s	−19.6 m	−19.6 m/s	−9.8 m/s²
3 s	−44.1 m	−29.4 m/s	−9.8 m/s²



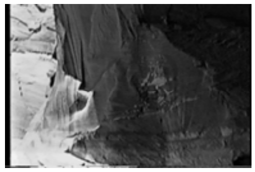
A more interesting example

How high was the cliff (what is d_V)?



A more interesting example?

How high was the cliff (what was d_V)?



t_{DOWN} = ~ 2 s, and using

$d_V = V_1t + ½at^2$, we learned that:

$d_V = 0 + ½ \cdot -9.81 m/s^2 \cdot 2s^2$, -19.6 m

Perhaps, more importantly, what was V_F?

$V_F = 0 + -9.81 m/s^2 \cdot 2s = 19.62$ m/s, or ~44 mph

What about horizontal motion of a projectile?

- Horizontal displacement or range of a projectile is the main index of performance in many cases of projectile motion
- Examples?

If air resistance is negligible, there is no net force in the horizontal direction ($\Sigma F = 0$); so, what is the horizontal acceleration?

Horizontal Motion of a Projectile

Given the equation: $d = v_1t + ½at^2$

Plus the knowledge that horizontal acceleration is zero, we can write:

$$d_H = v_Ht$$

specifically for horizontal (H) displacement

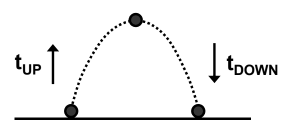
This allows us to calculate the range of a projectile, if we know the horizontal velocity and air time. We now know about velocity, but what about air time?

More about time of Flight

Determined by two factors:

- Vertical speed at release
- Height of release

Also, if takeoff height = landing height, then $t_{UP} = t_{DOWN}$

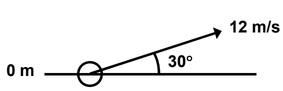


Some Practice: The Shot Put

How far will the shot travel (what is d_H)?

Case I

speed of release = 12 m/s
height of release = 0 m
angle of release = 30°



Some Practice: The Shot Put

Use $d_H = v_Ht$ need to know v_H and t

$v_H = v \cos \theta = 12$ m/s $\cdot \cos 30° = 10.4$ m/s

What is total time in the air (t_{TOT})?

If height of release = 0, then $t_{UP} = t_{DOWN}$ and $t_{TOT} = t_{UP} + t_{DOWN}$

What else do we know?

For upward part of flight:

$v_1 = v_V = v \cdot \sin \theta = 12$ m/s $\cdot \sin 30° = 6$ m/s

$v_2 = 0$ m/s $a = -9.81 m/s^2$

Some Practice: The Shot Put

Find appropriate equation for constant acceleration

$v_2 = v_1 + at$

Plug in v_1, v_2, and a, then solve for t:

$0\ m/s = 6\ m/s + (-9.81 m/s^2)(t_{UP})$

$t_{UP} = 0.61\ s$

t_{TOT} will be $2 \times t_{UP}$: $t_{TOT} = 2 \times 0.61\ s = 1.22\ s$ (Why ?)

$d_H = v_H\, t_{TOT} = 10.4\ m/s \times 1.22\ s = 12.7\ m$

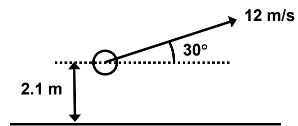
So, the shot traveled 12.7 m horizontally

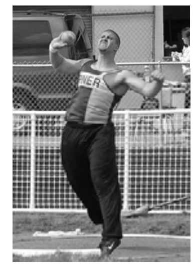
Some Practice: The Shot Put

Lets use a more realistic height of release

Case II

speed of release = 12 m/s
height of release = 2.1 m
angle of release = 30°





Some Practice: The Shot Put

We still use $d_H = v_H t$, but now $t_{UP} \neq t_{DOWN}$

v_H is found the same way as before:

$v_H = v \cos \theta = 12\ m/s \cdot \cos 30° = 10.4\ m/s$

and t_{UP} is also the same (using $v_2 = v_1 + at$):

$0\ m/s = 6\ m/s + (-9.81 m/s^2)(t_{UP})$

$t_{UP} = 0.61\ s$

So how do we calculate t_{DOWN}?

We need to find the upward (d_{UP}) and downward (d_{DOWN}) displacements

Some Practice: The Shot Put

Find another appropriate equation

$d = v_1 t + \frac{1}{2} a t^2$

Calculate the upwards displacement:

$d_{UP} = (6\ m/s)(0.61\ s) + \frac{1}{2}(-9.81\ m/s^2)(0.61\ s)^2 = 1.83\ m$

$d_{DOWN} = d_{UP}$ + ht of release $= 1.83\ m + 2.1\ m = 3.93\ m$

but this is in the negative direction, so it is −3.93 m

Now find t_{DOWN}:

$-3.93\ m = (0\ m/s)\, t_{DOWN} + \frac{1}{2}(-9.81\ m/s^2)(t_{DOWN})^2$

$0.8012\ s^2 = (t_{DOWN})^2$, so $t_{DOWN} = 0.90\ s$

Some Practice: The Shot Put

Now calcuate t_{TOT}

$t_{TOT} = t_{UP} + t_{DOWN} = 0.61\ s + 0.90\ s = 1.51\ s$

and finally, calculate the horizontal displacement:

$d_H = v_H\, t_{TOT} = 10.4\ m/s \times 1.51\ s = 15.7\ m$

So, now the shot travels 15.7 m horizontally

The increased height of release resulted in a 3.0 m (~20%) improvement in performance!

Some Application

Based upon what we have learned, what three primary factors can one manipulate to influence the horizontal displacement of a projectile?

Relative Height of Release

Speed of Release

Angle of Release

Some Application

Does increasing the height of release always lead to greater d_H?

Yes, but why?

$d_H = v_H \, t_{TOT}$

What about speed of release? Does increasing speed of release always lead to greater d_H?

Yes, but again, why?

$d_H = v_H \, t_{TOT}$

Some Application

What about angle of release?

Situation is more complicated, but…

When height of release is 0, optimal angle for maximum d_H is 45°

As height of release ↑, optimal angle ↓

Some Application

To which of the three factors (speed, angle, and height of release) is d_H most sensitive?

$d_H = v_H \, t_{TOT}$

Speed of release can positively affects v_H and t_{TOT}

What are the implications of this for training and/or coaching?

More Application…

How do actual angles of release in various sports compare to theoretical optimum?

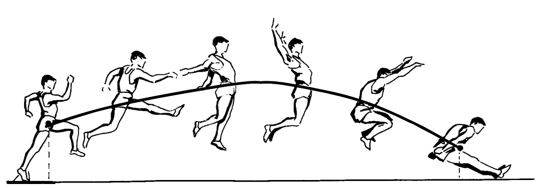
Shot Put:

- Positive height of release, so optimal angle should be slightly lower than 45°
- Theoretically optimal angle is about 40-41°
- Skilled shot-putters use angles of 35-37°
- Close, but why the discrepancy?

More Application…

Long Jump:

Positive height of release, so optimal angle should be slightly lower than 45°



Projectile Motion: Theory vs Reality

Long Jump:

- Theoretically optimal angle is about 42°
- Top long jumper use angles of 17-23°
- Way off! Why the major discrepancy?

When traveling at ~10 m/s, there is not enough time to generate a large takeoff angle

Long jumpers sacrifice optimal angle to get maximum speed. Does this seem like a good idea?

Projectile Motion: Summary

Objects (ball, javelin, human body) in the air are projectiles

Center of mass motion is determined by gravity and air resistance (which may be negligible)

Projectiles obey constant acceleration, making them easier to describe and understand (Galileo's equations)

Three factors determine trajectory, including horizontal displacement, of a projectile:

- Speed of release
- Angle of release
- Height of release