Beckham as physicist?

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Abstract

It is hard to think of a medium that does not use football or soccer as a means of promotion. It is also hard to think of a student who has not heard of David Beckham. If football captures the interest of students it can be used to teach physics; in this case a Beckham free-kick can be used to introduce concepts such as drag, the Bernoulli principle, Reynolds number and the Magnus effect, by asking the simple question: How does he curve the ball so much? Much basic mechanics can also be introduced along the way.

Figure 1. As David Beckham prepares to kick the ball he works out how to give the ball the precise spin, velocity and take-off angle needed to successfully outwit his opponents. (Photograph © Allsports UK.)

The Magnus effect

Consider a ball as shown in figure 2.

The air travels faster over the side of the ball where it is moving in the same direction as the flow of the air. The opposite is true of the other side of the ball. According to the Bernoulli principle [1] faster moving air reduces the pressure. A pressure difference is therefore set up on either side of the ball and this creates a net force, known as the Magnus effect. The force due to the Magnus effect, also referred to as the lift force, is given by

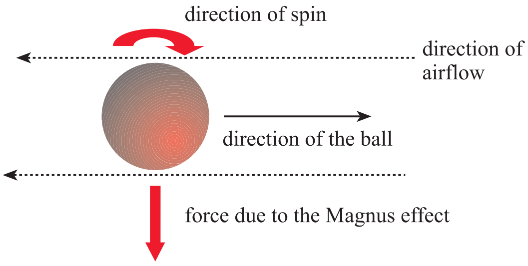

Figure 2. A spinning ball and the Magnus effect.

$$F_L = C_L \rho D^3 f v$$

where C_L is the lift coefficient, ρ is the density of the air (1.20 kg m^{-3} at sea level), D is the diameter of the ball (the world governing body, FIFA, state that the circumference of the ball must be between 0.68 and 0.70 m; we can therefore assume a diameter of 0.22 m), f is the spin frequency of the ball and v is the velocity of the ball.

The lift coefficient turns out to be a rather complex quantity which needs to be determined experimentally, but a figure of 1.23 meets the requirements of most sporting situations [2].

How much does, or can, Beckham swing the ball?

First we make the following assumptions (see figure 3): the free-kick is 25 m from the goal and is struck with a velocity of 25 m s^{-1} in such a way as to cause it to spin at 10 rev s^{-1}. The Magnus or

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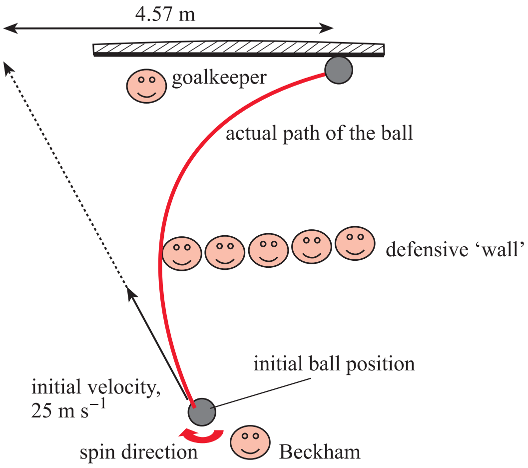

Figure 3. Beckham scores by the application of physics.

lift force can be calculated as

$$F_L = C_L \rho D^3 f v$$
$$= 1.23 \times 1.20 \times 0.22^3 \times 10 \times 25 = 3.93 \text{ N}.$$

This compares well with the figure given by Asai *et al* [3] for a Roberto Carlos free-kick during the 1998 World Cup.

Knowing the force on the ball, the acceleration can be calculated from $F = ma$. FIFA specify that at the start of a game the mass of a ball must be between 0.410 and 0.450 kg. If we assume the mean value of 0.430 kg then

$$F = ma \Rightarrow a = \frac{3.93 \text{ N}}{0.430 \text{ kg}} = 9.14 \text{ m s}^{-2}.$$

Assuming a time of flight of one second and applying $s = ut + \frac{1}{2}at^2$, the curve or swing can be calculated:

$$s = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2} \times 9.14 \times 1^2 = 4.57 \text{ m}.$$

The effect of drag

In the above example the effect of drag was omitted. Most students are aware of drag: try running waist deep in water or doubling your speed on a bicycle. The drag force will always act so as to oppose the motion of the ball; it slows it down. Unfortunately the effect of drag on a football is rather complicated and relies on empirical data.

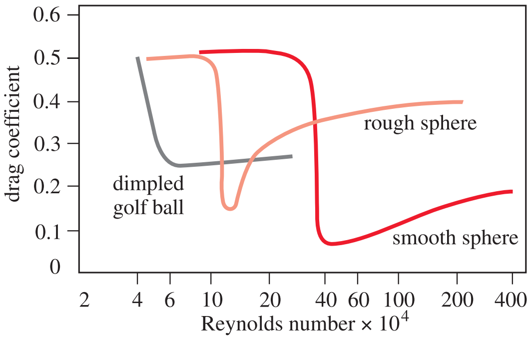

Figure 4. Variation of drag coefficient with Reynolds number for different spheres.

It can, however, be reduced to a simple (looking) formula:

$$F_d = \tfrac{1}{2} C_d \rho A v^2$$

where C_d is the the drag coefficient, ρ is the density of the air, A is the cross-sectional area of the ball (assume a diameter of 0.22 m and use $A = \pi D^2/4$) and v is the the velocity of the ball.

The problem is really with the drag coefficient, which varies, sometimes dramatically, with the velocity of the ball. Figure 4, adapted from Asai *et al* [3], presents data on the variation of drag with Reynolds number.

Reynolds number

Since it is not possible to derive the drag coefficient empirically for every new situation the Reynolds number allows the use of a graph as shown in figure 4. For practical purposes a football is a smooth sphere and the Reynolds number can be calculated from

$$Re = \rho v D / \eta$$

where η is the viscosity of the air. Daish [4] gives a simplified version, which works well over a wide range, of the Reynolds number calculation in air as

$$Re = 640 v D.$$

The viscosity of air at 20 °C can be taken to be 18.2 μPa s, which for a ball with initial velocity 25 m s^{-1} gives a Reynolds number of

$$Re = \frac{\rho v D}{\eta} = \frac{1.20 \times 25 \times 0.22}{18.2 \times 10^{-6}} = 3.63 \times 10^5$$

Table 1. Reynolds number, drag coefficient and drag force at various velocities.

Velocity v (m s^{-1})	Reynolds number Re ($\times 10^4$)	Drag coefficient C_d	Drag force F_d (N)
0	0	0	0
2.5	3.63	0.50	0.1
5.0	7.25	0.50	0.3
7.5	10.9	0.50	0.6
10.0	14.5	0.50	1.1
12.5	18.1	0.50	1.8
15.0	21.8	0.50	2.6
17.5	25.4	0.50	3.5
20.0	29.0	0.45	4.1
22.5	32.6	0.40	4.6
25.0	36.3	0.20	2.9
27.5	39.9	0.10	1.7
30.0	43.5	0.06	1.2

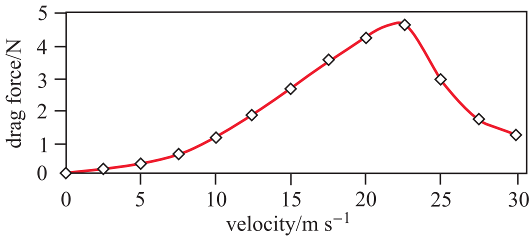

Figure 5. Variation of drag force with velocity.

which from figure 4 gives an estimate of the order of 0.1 for the drag coefficient.

The drag force can then be calculated to be

$$F_d = \tfrac{1}{2} C_d \rho A v^2$$
$$= \tfrac{1}{2} \times 0.1 \times 1.2 \times (\pi D^2/4) \times 25^2 = 2.85 \text{ N}.$$

Table 1 gives the Reynolds number, Re, the estimated drag coefficient, C_d, and the drag force acting on a soccer ball at different velocities. These data are plotted in figure 5.

What these data show is that if a player can hit the ball hard enough, 25–30 m s^{-1}, then not only will the ball travel faster initially but it will also maintain its velocity for longer than a slower ball. It is also apparent that as a ball slows, on its path to goal, the drag force is going to increase, which will exaggerate to swing due to the Magnus effect.

So how does he kick the ball so hard?

A study of the mechanics involved in the collision between the foot and the ball suggests that the velocity of the ball is dependent on the mass of the player's leg and foot, the mass of the ball and the coefficient of restitution. Plagenhoef [5] presents the following relationship:

$$v_{\text{ball}} = v_{\text{foot}} \frac{M(1+e)}{M+m}$$

where v is the velocity, M is the mass of leg and foot and m is the mass of ball. Reilly [6] estimates that 'realistic' data give $M/(M+m) = 0.8$ and $e = 0.5$. This being the case $1 + e = 1.5$, which allows for a simplified equation for the ball velocity:

$$v_{\text{ball}} = 1.2 v_{\text{foot}}.$$

This being the case Beckham would need to hit the ball with a foot velocity of 20.8 m s^{-1} to impart a velocity of 25 m s^{-1}.

Projectile motion

Rather than swing the ball around a defensive wall the kicker may elect to 'chip' the ball over the wall. If the ball is chipped over the defensive wall it can be treated as a 'projectile motion' and the following simple equations (see figure 6) can be applied.

Range, $R = v_0 t \cos\theta$
Height, $H = v_0 t \sin\theta - \tfrac{1}{2} g t^2$
Time of flight, $t_f = (2 v_0 \sin\theta)/g$
Maximum range, $R_0 = (v_0 \sin 2\theta)/g$
Maximum height, $H_0 = (v_0^2 \sin^2\theta)/2g$

where t is time and it is assumed that when $t = 0$ both H and R are zero.

Unfortunately this analysis assumes that the drag force can be neglected. Having discussed the effect of drag we know that this is not the case and the question arises as to how to take drag into account.

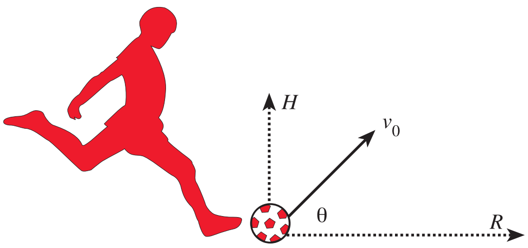

Figure 6. Treating the free-kick as a projectile motion.

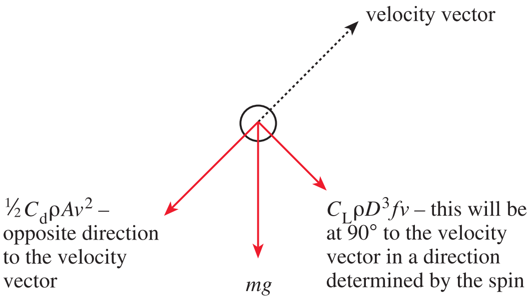

Figure 7. Force diagram for the free-kick.

Projectile motion and drag

The projectile can be considered to have three forces acting on it, its weight mg, the Magnus force, if spinning, $C_L\rho D^3 f v$ and the drag force, $\frac{1}{2}C_d\rho A v^2$. A first approach can be to draw a force diagram as in figure 7.

The resultant force vector will give the direction, and the magnitude can be calculated either from the scale drawing or, following Carini [7], the acceleration, a, can be estimated from

$$a/g = F_{\text{resultant}}/\text{weight}.$$

This stems from $F_{\text{resultant}} = ma$ and weight $= mg$. A more detailed analysis, beyond the scope of this paper, would require the solving of $\mathrm{d}r/\mathrm{d}t$ for velocity and $\mathrm{d}^2r/\mathrm{d}t^2$ for acceleration, where r is the position vector $r \equiv (x, y, z)$ taking x, y and z as the usual spatial coordinates.

Full time

In conclusion it is hoped that this paper introduces an alternative approach to teaching a variety of physics topics which many students are likely to find interesting. More advanced students may wish to explore the physics of soccer or other sports to a greater depth and it is hoped that this paper can provide a springboard for such endeavours.

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