Constant Acceleration

Galileo's equations of constant acceleration:

$$v_2 = v_1 + at$$
$$d = v_1 t + \tfrac{1}{2}at^2$$
$$v_2^2 = v_1^2 + 2ad$$

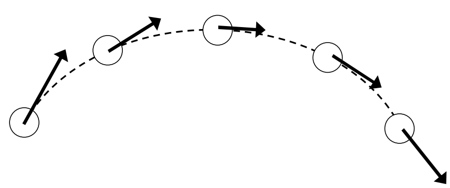
d = displacement
v = velocity
a = acceleration
t = time
subscript 1 = initial time
subscript 2 = final time

Projectile Motion

- **A projectile is an object that is in free fall and is subject only to gravity and air resistance**
- **Often (long and high jump, shot put, free throw), air resistance is small and we ignore it**
- **Often, (golf or tennis) air resistance should be considered**
- **If air resistance is ignored, projectiles are an example of constant acceleration**

Projectile Motion

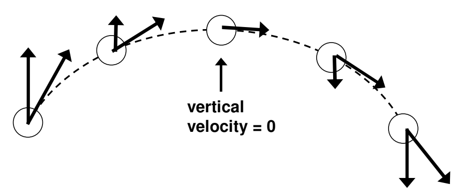
- **If air resistance is negligible, projectile motion is simply a special case of uniform acceleration**
- **With no air resistance, the path followed by a projectile will be a <u>parabola</u>**



Vertical Velocity

Horizontal and vertical components of velocity are independent, and treated separately

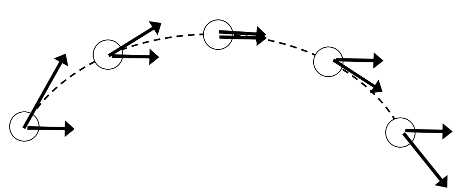
Gravity will cause the vertical component of velocity to change during the flight



Horizontal Velocity

The horizontal component of velocity will be constant for the whole flight. But, why?

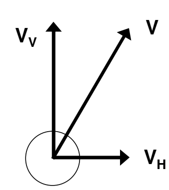
Because there are no considerable forces acting on the object, in the horizontal direction.



Projectile Motion

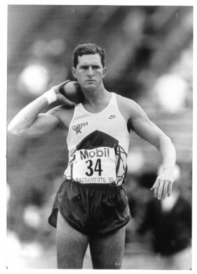
Thus, if air resistance is negligible:

- **Horizontal velocity does not change while the object is in the air**
- **Vertical velocity changes by -9.81 m/s, each second the object is in the air; how else might we phrase this?**



Projectile Trajectory

To influence the trajectory of a projectile, we control 3 factors:



Speed of Release

Angle of Release

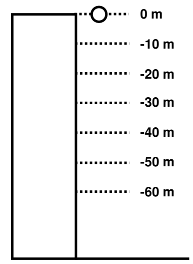
Relative Height of Release

Vertical Motion of a Projectile

Drop a water balloon from the top of a building, and the ballon's vertical motion will be very predictable

Use: $d = v_1 t + ½at^2$
$v_2 = v_1 + at$

time	position	velocity	acceleration
0 s			
1 s			
2 s			
3 s			



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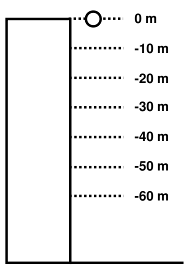
Use: $d = v_1 t + ½at^2$
$v_2 = v_1 + at$

time	position	velocity	acceleration
0 s	0 m	0 m/s	–9.8 m/s²
1 s			
2 s			
3 s			



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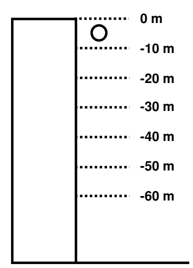
time	position	velocity	acceleration
0 s	0 m	0 m/s	–9.8 m/s²
1 s	–4.9 m	–9.8 m/s	–9.8 m/s²
2 s			
3 s			



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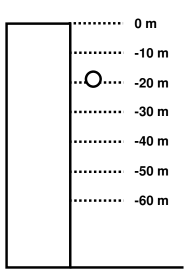
Use: $d = v_1 t + ½at^2$
$v_2 = v_1 + at$

time	position	velocity	acceleration
0 s	0 m	0 m/s	–9.8 m/s²
1 s	–4.9 m	–9.8 m/s	–9.8 m/s²
2 s	–19.6 m	–19.6 m/s	–9.8 m/s²
3 s			



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Use: $d = v_1 t + ½at^2$
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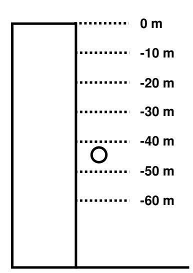
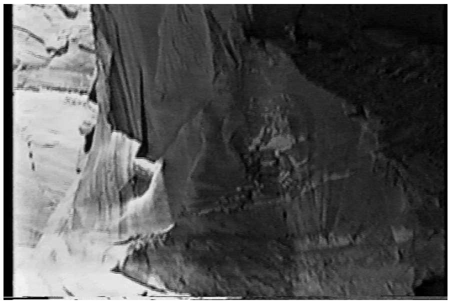
time	position	velocity	acceleration
0 s	0 m	0 m/s	–9.8 m/s²
1 s	–4.9 m	–9.8 m/s	–9.8 m/s²
2 s	–19.6 m	–19.6 m/s	–9.8 m/s²
3 s	–44.1 m	–29.4 m/s	–9.8 m/s²



A more practical example?

How high was the cliff (what is d_V)?



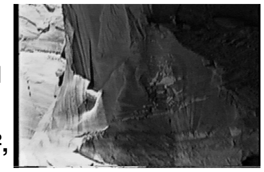
A more practical example?

How high was the cliff (what was d_V)?

t_{DOWN} = ~ 2 s, and using

$d_V = V_I t + ½at^2$, we learned that:

$d_V = 0 + ½ \cdot -9.81 m/s^2 \cdot 2s^2$, -19.6 m



Perhaps, more importantly, we learned that the $V_F = 0 + -9.81 m/s^2 \cdot 2s = 19.62$ m/s, or ~44 mph

Horizontal Motion of a Projectile

- Horizontal displacement or range of a projectile is the main index of performance in many cases of projectile motion
- Examples?

If air resistance is negligible, there is no net force in the horizontal direction ($\Sigma F = 0$): what will the horizontal acceleration be?

Horizontal Motion of a Projectile

Given the equation: $d = v_1 t + ½at^2$

Plus the knowledge that horizontal acceleration is zero, we can write:

$$d_H = v_H t$$

specifically for horizontal (H) displacement

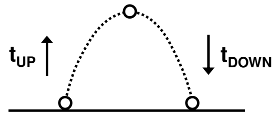
This allows us to calculate the range of a projectile, if we know the horizontal velocity and the time in the air (how do we find time in the air?)

Time of Flight,

is determined by two factors:

- Vertical speed at release
- Height of release

If takeoff vertical position = landing vertical position, then $t_{UP} = t_{DOWN}$



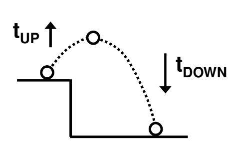
Time of Flight

But what if:

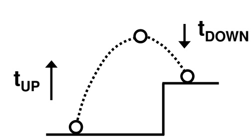
takeoff vertical position ≠ landing vertical position?

Two possibilities:

$t_{UP} < t_{DOWN}$



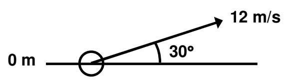
$t_{UP} > t_{DOWN}$



The Shot Put

How far will the shot travel (what is d_H)?

speed of release = 12 m/s
height of release = 0 m
angle of release = 30°



The Shot Put

Use $d_H = v_H t$ need to know v_H and t

$v_H = v \cos \theta = 12$ m/s · cos 30° = 10.4 m/s

What is total time in the air (t_{TOT})?

If height of release = 0, then $t_{UP} = t_{DOWN}$
and $t_{TOT} = t_{UP} + t_{DOWN}$

What else do we know?

For upward part of flight:

$v_1 = v_V = v \cdot \sin \theta = 12$ m/s · sin 30° = 6 m/s

$v_2 = 0$ m/s $a = -9.81$ m/s^2

The Shot Put

Find appropriate equation for constant acceleration

$v_2 = v_1 + at$

Plug in v_1, v_2, and a, then solve for t:

0 m/s = 6 m/s + (–9.81m/s^2)(t_{UP})

$t_{UP} = 0.61$ s

t_{TOT} will be $2 \times t_{UP}$: $t_{TOT} = 2 \times 0.61$ s = 1.22 s (Why ?)

$d_H = v_H t_{TOT} = 10.4$ m/s $\times$ 1.22 s = 12.7 m

So, the shot traveled 12.7 m horizontally

Projectile Motion

Does increasing the height of release always lead to greater d_H?

Yes, but why?

$d_H = v_H t_{TOT}$

What about speed of release?

Does increasing speed of release always lead to greater d_H?

Yes, but again, why?

$d_H = v_H t_{TOT}$

Projectile Motion

What about angle of release?

Situation is more complicated, but…

When height of release is 0, optimal angle for maximum d_H is 45°

As height of release ↑, optimal angle ↓

For a given positive height of release, optimal angle shifts back towards 45° as speed of release increases

Projectile Motion

To which of the three factors (speed, angle, and height of release) is d_H most sensitive?

$d_H = v_H t_{TOT}$

Speed of release: affects both v_H and t_{TOT}

What are the implications of this for training and/or coaching?

Projectile Motion: Theory vs Reality

How do actual angles of release in various sports compare to theoretical optima?

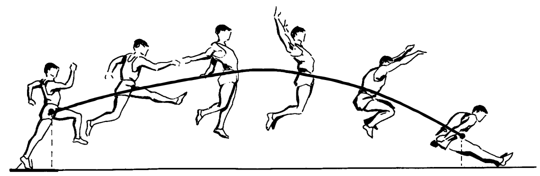
Shot Put:

- **Positive height of release, so optimal angle should be slightly lower than 45°**
- **Theoretically optimal angle is about 40-41°**
- **Skilled shot-putters use angles of 36-37°**
- **Close, but why the discrepancy?**

Projectile Motion: Theory vs Reality

Long Jump:

Positive height of release, so optimal angle should be slightly lower than 45°



Projectile Motion: Theory vs Reality

Long Jump:

- **Theoretically optimal angle is about 42°**
- **Top long jumper use angles of 17-23°**
- **Way off! Why the major discrepancy?**

When traveling at ~10 m/s, there is not enough time to generate a large takeoff angle

Long jumpers sacrifice optimal angle to get maximum speed. Does this seem like a good idea?

Projectile Motion: Summary

Objects (e.g., ball, javelin, human body) that are in the air are projectiles

Motion is completely determined by gravity and air resistance (which may be negligible)

Three factors determine the trajectory of a projectile:

Speed of release
Angle of release
Height of release